N71-17572

Report No. S-70-5

NASA CR-116504

RESIDUAL STRESSES DEVELOPED IN MOLDING A FLAT SHEET OF A CHEMICALLY-HARDENING RELATIVELY INCOMPRESSIBLE MATERIAL

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NGL-33-016-067

September 1970

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School of Engineering and Science

University Heights, New York, N.Y. 10453

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NEW YORK UNIVERSITY
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Abstract

Residual thermal stresses in a flat sheet molded from a chemically hardening material are approximated by an expression having a simple closed form representation whenever the shear modulus is much smaller than the bulk modulus in the fully hardened material. The result is used to evaluate the hardening rate which yields maximum residual stresses in the interior of a sheet. A comparison between numerical results obtained from the exact and approximate solutions to the problem show that the more easily obtained latter results represent a conservative estimate of the exact stress distribution.

Introduction

As part of a current investigation devoted to the determination of residual thermal stresses in material undergoing an exothermic reaction,[1,2]* it was found that during the molding process the sole non-vanishing residual stress component is parallel to the wall surface. Unfortunately, the mathematical expression for this stress component took the form of an infinite series in which the coefficients themselves were infinite series. Although a solution can be readily obtained in numerical form, the expression is not amenable to further investigation in an analytical form.

It is observed that a useful mathematical simplification is often made in studying the behavior of time dependent materials by considering the material to be incompressible.[3,4,5,] Perhaps the same advantage can be obtained in the investigation of the molding process. However, instead of starting the

*Superscript numbers in squared brackets refer to the references listed in the Bibliography.

analysis under the assumption of an incompressible material a comparable result may be obtained by viewing the incompressible material as the limit of a material whose shear modulus is much smaller than its bulk modulus, in which case use can be made of the previously derived solution. Furthermore, it will be shown that the expression developed for the residual stresses in the wall may then be approximated by an equation which can be written in closed form in terms of elementary functions.

It is noted, of course, that in materials for which the shear modulus is much smaller than the bulk modulus, Poisson's ratio approaches one half. The limiting value is the same as Poisson's ratio in an incompressible material.

Reduction of the Residual Stress to Closed Form

It has been shown[2] that when a material is poured or injected between two parallel planes which comprise the mold, and hardens as the result of an exothermic chemical reaction while the mold surfaces are maintained at the initial ambient temperature, the residual stress component parallel to the wall surface is given by the expression

$$\sigma_P = -6K\alpha'\left(H/\rho c\right)\left(\frac{1+\mu}{1-\mu}\right)\sum_n^{\infty}\left[\sum_m^{\infty}\frac{m\eta^m}{(m+1)\left[m^2+\left(\frac{n\pi}{\nu}\right)^2\right]}\right]\frac{2-n\pi\sin n\pi x}{(n\pi)^2} \tag{1}$$

$$n=1,3,5\ldots \quad , \quad m=1,2,3,\ldots$$

where the parameter η was defined

$$\eta = \frac{4G_o}{3K+4G_o} \tag{2}$$

or as a function of Poisson's Ratio μ measured in the fully hardened state

$$\eta = \frac{2}{3}\left[\frac{1 - 2\mu}{1 - \mu}\right] \tag{3}$$

The bulk modulus is K, G_o is the shear modulus of the fully hardened material, and α' is the coefficient of thermal expansion. The term ν^2 is called the reaction rate parameter and the factor $H/\rho c$ measures the maximum possible rise in temperature of the reacting material in terms of the volumetric heat of reaction H, the material density ρ, and the specific heat c. The x coordinate is measured from one of the surfaces of the sheet whose thickness has been normalized to unity.

The ratio of shear modulus to bulk modulus may be expressed[6] as

$$\frac{G_o}{K} = \frac{3(1-2\mu)}{2(1+\mu)} \tag{4}$$

When G_o is small compared to K, Poisson's ratio μ tends towards one-half, and the parameter η approaches zero. Under these circumstances, the internal series of Equation (1) may be simplified by retaining only the first term, and neglecting higher powers of η, so that expression for residual stresses can be written

$$\sigma_p = -3K\alpha'\left(H/\rho c\right)\left(\frac{1+\mu}{1-\mu}\right)\eta \sum_{n}^{\infty} \frac{2 - n\pi \sin n\pi x}{(n\pi)^2[1+(n\pi/\nu)^2]} \quad n=1,3,5\ldots \tag{5}$$

The quantities $(1+\mu)/(1-\mu)$ and η are shown by Equations (2) and (3) to be expressible in terms of G_o and K, so that Equation (5) may be written

$$\sigma_P = -3K\alpha'(H/\rho c)\left[\frac{9K}{3K+4G_o}\right]\left[\frac{4G_o}{3K+4G_o}\right]\sum_n^{\infty}\frac{2 - n\pi\sin n\pi x}{(n\pi)^2[1+(n\pi/\nu)^2]} \qquad (6)$$

$$n = 1,3,5\ldots$$

Under the condition that $G_o/K \ll 1$,

$$\frac{9K}{3K + 4G_o} = \frac{3}{1 + 4G_o/3K} \approx 3\,(1 - 4G_o/3K\ldots.) \qquad (7)$$

and

$$\frac{4G_o}{3K + 4G_o} = \frac{4G_o}{3K}\left(\frac{1}{1 + 4G_o/3K}\right) \approx \frac{4G_o}{3K}\left(1 - 4G_o/3K\ldots.\right) \qquad (8)$$

Multiplication of the preceding quantities, and retention of the linear term in the ratio G_o/K shows that Equation (6) can be expressed as

$$\sigma_P = 12\,G_o\alpha'(H/\rho c)\sum_n^{\infty}\left[\frac{\nu^2\sin n\pi x}{n\pi[\nu^2+(n\pi)^2]} - \frac{2\nu^2}{(n\pi)^2[\nu^2+(n\pi)^2]}\right] \qquad (9)$$

$$n = 1,3,5\ldots.$$

Elsewhere it has been shown from a study of the Fourier Series representation of some hyperbolic functions,[7,8] that

$$\sum_n^{\infty}\frac{\sin n\,\theta}{n(a^2+n^2)} = \frac{\pi}{4a^2}\left[1 - \frac{\cosh\,[a(\pi/2-\theta)]}{\cosh\,a\pi/2}\right] \quad 0 \le \theta \le \pi \qquad (10)$$

$$n = 1,3,5\ldots.$$

and

$$\sum_n^{\infty}\frac{1}{n^2(a^2+n^2)} = \frac{\pi}{4a^2}\left[\frac{\pi}{2} - \frac{1}{a}\tanh\frac{a\pi}{2}\right], \quad n = 1,3,5\ldots. \qquad (11)$$

Comparison of the preceding equations with Equation (9) indicates that the two series which appear in Equation (9) can be written in terms of hyperbolic functions. The result of this substitution permits us to put the expression for residual stresses into the form

$$\sigma_p = 3G_o\alpha'(H/\rho c)\left[\frac{2}{\nu}\tanh\frac{\nu}{2} - \frac{\cosh[\nu(1/2-x)]}{\cosh\nu/2}\right] \tag{12}$$

The latter expression is applicable as long as the ratio G_o/K is much less than unity.

Discussion of the Results

One instance of the advantage of the approximate solution in closed form, obtained when the ratio of shear to bulk modulus is much less than unity, arises when the application requires further mathematical analysis, a task not easily accomplished with the complete solution.

It was observed in the discussion of the numerical results portion of a previous paper[2] that the residual stress component parallel to the wall surface of the material being molded reaches a maximum tensile value at the center of the sheet at a rate of hardening which is as yet undetermined but can be measured by the reaction rate parameter ν^2. Such a state of stress may either be desirable or undesirable, depending upon the ultimate use of the slab being molded. The hardening rate which produces the largest tensile stresses can of course be evaluated from the exact solution numerically. It is much simpler, however, to estimate the value of the reaction rate parameter at which the maximum tensile stress in the interior from the newly obtained approximate solution.

At the center plane of the sheet $x = 1/2$, where the maximum tensile stress is known to occur, Equation (12) shows that the residual stress component parallel to the mold surfaces is given by the expression

$$\sigma_{P}\Big|_{x=1/2} = 3\alpha' G_o (H/\rho c)\left[\frac{2}{\nu} \tanh \frac{\nu}{2} - \operatorname{sech} \frac{\nu}{2}\right] \qquad (13)$$

In order to determine the reaction rate parameter ν , which results in the maximum tensile stress, take the derivative of the preceding expression with respect to ν , and set the result equal to zero to find

$$0 = 3\alpha' G_o (H/\rho c)\left[-\frac{2}{\nu^2} \tanh \frac{\nu}{2} + \frac{1}{\nu} \operatorname{sech}^2 \frac{\nu}{2} + \frac{1}{2} \sinh \frac{\nu}{2} \operatorname{sech}^2 \frac{\nu}{2}\right] \qquad (14)$$

The desired reaction rate parameter requires the solution of the transcendental equation

$$1 + \frac{\nu}{2} \sinh \frac{\nu}{2} - \frac{1}{\nu} \sinh \nu = 0 \qquad (15)$$

It can be found numerically that ν approximately equal to 5.5, or $\nu^2 = 30$ satisfies the previous equation for ν , and represents the reaction rate parameter which yields a maximum residual tensile stress at the center plane.

It was previously shown[1] that the parameter ν^2 is equal to $Q_o L^2/\kappa H$, where Q_o denotes the initial rate of volumetric heat generation in the hardening material, L is the thickness of the wall, κ is the thermal diffusivity of the wall material, and H is the volumetric heat of reaction. Thus it is seen that the value of ν^2 is determined not only by the intrinsic speed of the reaction, but also by the thermal properties and dimensions of the slab. Hence adjustment of the reaction rate parameter in order to alter the tensile residual thermal stress may be accomplished by changing either the chemical constants, the thermal constants, or the wall thickness separately or in a suitable combination.

The accuracy of Equation (12) as an approximation to the exact series solution of Equation (1) may be seen by the comparison of Fig. 1, where stress represented in dimensionless form by the variable $\sigma/\alpha'(H/\rho c)K$ is shown as a function of x/L, the dimensionless coordinate measured from one surface of the sheet for two representative values of the reaction rate parameter. It is observed that the accuracy of the closed form approximation decreases as the ratio of shear to bulk modulus increases, which is to be expected. The two solutions are in fair qualitative agreement when G_o/K is approximately one-fifth. It is also observed that the magnitude of the stress variable determined by the approximate relation of Equation (12) exceeds that given by the exact solution of Equation (1), so that the closed form approximation turns out to provide a conservative estimate of the residual stresses.

Conclusion

It has been shown that the previously derived expression for the residual stress distribution which arises in the molding of a slab from an exothermic chemically hardening material may be approximated by a simple closed form result when the ratio of shear to bulk modulus of the material G_o/K is small. For descriptive purposes such a material has been called approximately incompressive because Poisson's Ratio is then in the neighborhood of one-half. Such properties are characteristic of many thermosetting plastics.

The closed form is found to give approximate results that are easily determined and represent a conservative estimate of the actual residual stress distribution.

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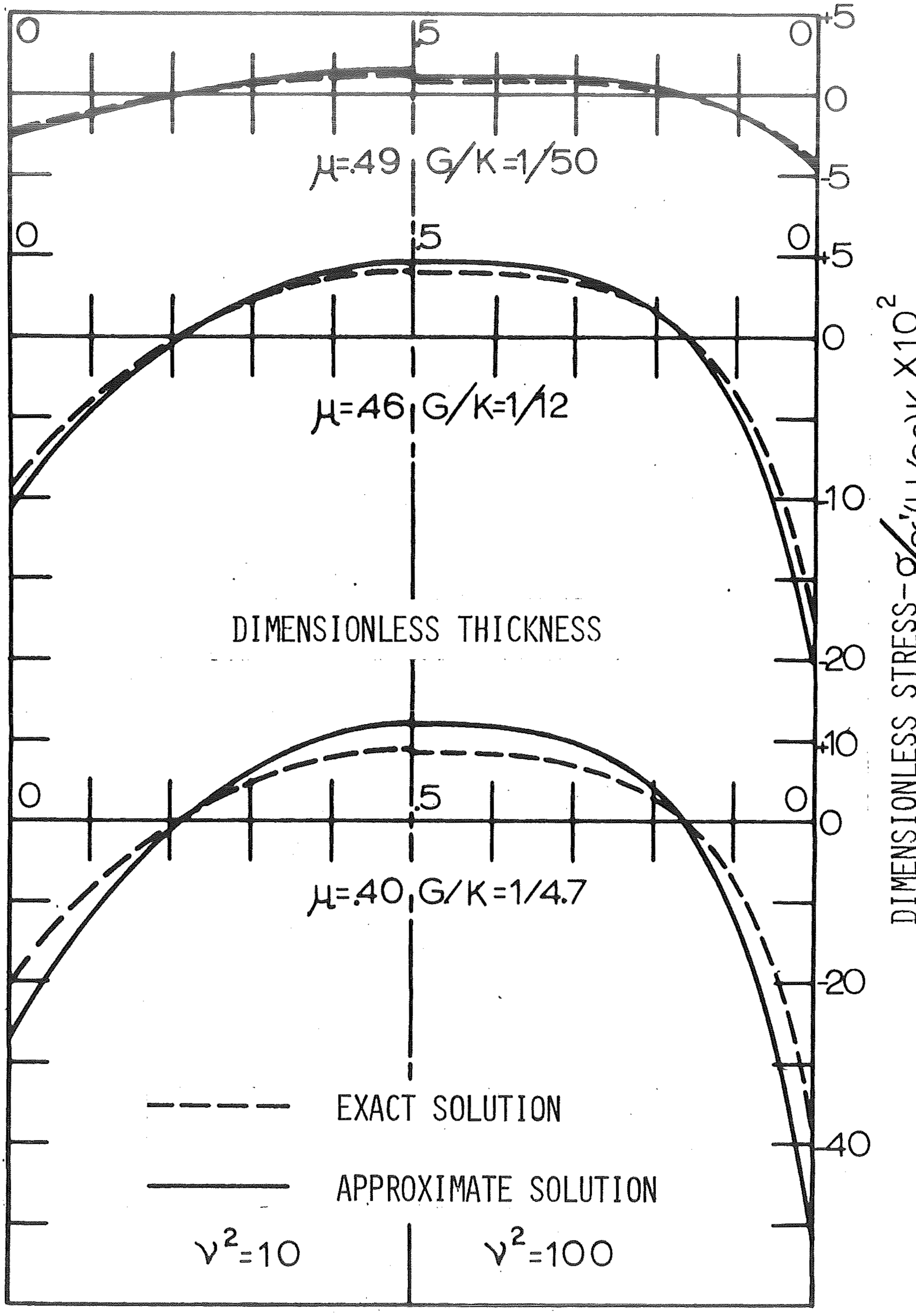

Fig. 1

A Comparison of Exact and Approximate Solutions for the Distribution of Residual Thermal Stress.